An allosteric model of calmodulin explains differential activation of PP2B and CaMKII

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Calmodulin plays a vital role in mediating bidirectional synaptic plasticity by activating either calcium/calmodulin-dependent protein kinase II (CaMKII) or protein phosphatase 2B (PP2B) at different calcium concentrations. We propose an allosteric model for calmodulin activation, in which binding to calcium facilitates the transition between a low-affinity [tense (T)] and a high-affinity [relaxed (R)] state. The four calcium-binding sites are assumed to be nonidentical. The model is consistent with previously reported experimental data for calcium binding to calmodulin. It also accounts for known properties of calmodulin that have been difficult to model so far, including the activity of nonsaturated forms of calmodulin (we predict the existence of open conformations in the absence of calcium), an increase in calcium affinity once calmodulin is bound to a target, and the differential activation of CaMKII and PP2B depending on calcium concentration.

Allosteric Model of Calmodulin. In our model, calmodulin can exist in two different states, the active open [relaxed (R)] state and the inactive closed [tense (T)] state. Each of these states can bind four calcium ions (Fig. 1). When no calcium is bound, the T state prevails, because its free energy is lower than that of the unbound R state. Consecutive binding of calcium ions, however, progressively stabilizes the R state until the free energy of the R state is lower than that of the T state, so the R state is favored.

The four different binding sites are designated A, B, C, and D (A and B on the N-terminal domain, C and D on the C-terminal domain, with no sequential order being implied within the domains). Each of the states and each of the reactions is explicitly modeled, with distinct dissociation constants and R to T transition probabilities for each of the sites (Fig. 2). The constant L describes the equilibrium between both states when no calcium ion is bound: \( L = [T]/[R] \). If L is very large, most of the protein exists in the tense state in the absence of calcium. If L is small (close to one), the R state is nearly as populated as the T state. The constants \( c_A, c_B, c_C, \) and \( c_D \) describe the ratio of dissociation constants for the R and T states for each site: \( c_i = K_{iR}/K_{iT} \). If c is one, both R and T states have the same affinity for calcium. The c values also indicate how much the equilibrium between T and R states changes upon calcium binding: the smaller c, the more the equilibrium shifts toward the R state.

The formula for fractional occupation of an allosteric protein in the absence of allosteric effectors (21) can be generalized to describe nonequivalent calcium-binding sites. In the case of four binding sites, the generalized expression is:

\[
\hat{Y} = 0.25 \sum_i \left( \alpha_i \prod_j (1 + \alpha_j) \right) + L \sum_i \left( c_i \alpha_i \prod_j (1 + c_i \alpha_j) \right)
\]

[1]

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Data deposition: The model reported in this paper has been deposited in the BioModels database, www.ebi.ac.uk/biomodels (accession code MODEL1988598404).

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**Parameter Determination.** The allosteric isomerization constant, $L$, the microscopic dissociation constants $K_R^a$ for the $R$ state for each binding site, and the ratios of $R$ and $T$ state affinity, $c_i$, for each site, cannot be directly obtained from experimental literature. Experimental results do allow us, however, to constrain parameter space and obtain reasonable estimates. We used calcium-binding data in the presence and absence of an allosteric activator for mutant and wild-type forms of calmodulin.

We introduced the additional assumption that all four $c_i$ values are identical. This is based on the fact that both $R$ and $T$ states are symmetrical and on the idea that the free energy of transition is spread over the whole molecule.

To obtain an estimate of $c$ and of the isomerization constant $L$, a reduced model of calmodulin was used. In this simple model, all four calcium-binding sites are equivalent, and one molecule of allosteric activator can bind to each molecule of calmodulin. In this case, the fractional occupation $Y$ of an allosteric protein in the presence of an allosteric activator can be described as (24):

$$Y = \frac{\alpha (1 + \alpha)^3 + L \left( \frac{1 + \alpha}{1 + \gamma} \right) c a (1 + c a)^3}{(1 + \alpha)^4 + L \left( \frac{1 + \alpha}{1 + \gamma} \right) (1 + c a)^4}.$$  

In this equation, $\alpha = [\text{Ca}^{2+}] / K_R$, $\gamma = [A] / K_{AR}$, and $c = K_{AR} / K_{AT}$; $K_R$ denotes the dissociation constant for calcium binding to the $R$ state, $L$ the allosteric equilibrium constant, $[A]$ the concentration of allosteric activator, $K_{AR}$ the dissociation constant for binding of the allosteric activator to the $R$ state, and $K_{AT}$ the dissociation constant for binding of the allosteric activator to the $T$ state. The factor $(1 + \gamma c)/(1 + \gamma)$ modulates the apparent isomerization constant as a function of activator concentration: if more target is present, the apparent value of $L$ decreases. Peersen et al. (25) have measured calcium binding to calmodulin in the absence and presence of several target proteins, which act as allosteric effectors of calmodulin. By taking the calcium concentration at $Y = 1/2$ from four different binding curves (absence of target, presence of skMLCK, PhK5, or CaATPase, respectively) and inserting them into the above equation, we obtained a system of four equations, which depend on $L$, $c$, and on $\gamma$ values for skeletal myosin light chain kinase, phosphorylase kinase, and Ca$^{2+}$-ATPase, respectively. We minimized this system of equations by using the least-square function provided in Scilab (www.scilab.org). To avoid local minima, we ran $10^5$ minimizations, each one with different initial values. By this approach we estimated $L$ to be 20,670 and $c$ to be $3.96 \times 10^{-3}$. A full list of equations and details about the minimization process are given in supporting information (SI).

To determine the dissociation constants for all four binding sites, we used a similar approach, making use of the general formula for fractional occupancy presented above (Eq. 1). The formula can be simplified to reflect calmodulin binding to calcium under different experimental constraints. For instance, it can be reduced to the two N-terminal binding sites to describe recombinant versions of calmodulin in which the C-terminal binding sites are ablated. Using the available experimental literature, one thus obtains a system of four equations and corresponding data: the full equation for $Y$ (using data from ref. 26), two reduced equations for $Y$ with N- or C-terminal binding sites only, respectively (using data from experiments with recombinant calmodulin from ref. 18) and one reduced equation.
for $\gamma$ in which only the $R$ state is considered (using data from ref. 25 in the presence of skMLCK, where it can be assumed that calmodulin exists mostly in the $R$ state). There are thus four constraints that can be used to restrict parameter space. We used the calcium concentration at $\gamma = \frac{1}{2}$ and at $\gamma = \frac{1}{4}$ from the experimental literature to obtain eight equations that depend on $K_i$ values. The eight equations are listed in SI.

To avoid the problem of local minima, we resorted to systematic sampling of parameter space and generated a script that tested all possible combinations of $K_i$ values within a broad range. A detailed description of this can be found in SI. The resulting values are $K_{a1} = 8.32 \times 10^{-6}$ M, $K_{a2} = 1.66 \times 10^{-8}$ M, $K_{a3} = 1.74 \times 10^{-5}$ M, and $K_{a4} = 1.45 \times 10^{-8}$ M.

**Kinetic Simulations.** Although Eq. 1 provides a general formula for calcium saturation in the case of four nonequivalent binding sites in the absence of allosteric effectors, the situation becomes less tractable in the presence of two competing allosteric activators (CaMKII and PP2B, in this case), especially at low calcium concentrations, where initial calcium concentration can differ significantly from free calcium concentration at steady state. Moreover, our intention was to provide an accurate model of the binding and dissociation of PP2B, and 128 reactions describing the binding and dissociation of CaMKII, another 32 reactions describing the corresponding dissociation events, and 128 reactions describing calcium binding to the target-free state. The full model comprises 352 reactions: 32 reactions describing the system of reactions for kinetic simulations.

Every reaction was split into a separate forward and backward reaction. The full model comprises 352 reactions: 32 reactions describing calcium binding to the target-free state, 32 reactions describing the corresponding dissociation events, 64 reactions describing calcium binding to and dissociation from the target-free state, 32 reactions describing transitions between the $R$ and $T$ states, 32 reactions describing the binding and dissociation of CaMKII, another 32 reactions describing the binding and dissociation of PP2B, and 128 reactions describing calcium binding to and dissociation from CaMKII- or PP2B-bound calmodulin. A complete list of reactions can be found in SI.

The $K_{a1}$ for calcium binding was assumed to be the same for all four binding sites and both states, because it can be assumed to be controlled only by calcium and calmodulin diffusion and size (random exploration). Different affinities were represented by different $K_{off}$ values.

Simulations were run by using the parameter-scan facility of the simulator COPAS (27).

**Comparison of Calcium-Binding Curve to Experimental Results.** Our model reproduces experimental measurements of calmodulin binding to calcium. Fig. 3 compares the outcome of our simulation with results reported by Crouch and Klee (16). [For more figures comparing the simulation outcome with data reported by Peersen et al. (25) and Porumb (30), refer to SI.]

Apparent sequential association constants obtained by fitting a version of the Adair equation (43) to our model are listed in Table 1. A comparison to association constants reported in the experimental literature about calmodulin (16, 28–30) shows that our association constants lie within the range of experimentally determined values.

**Activity of Various Forms of Nonsaturated Calmodulin.** By calculating the equilibrium constant for the transition between $R$ and $T$ states for calmodulin species that are bound to one or more calcium ions, one can see what fraction of calmodulin is active under steady-state conditions.

For instance, the relation between the $R$ and $T$ states for calmodulin with exactly two calcium ions bound is given by $(R/T) = 1/(Lc^2 \approx 3)$. Similarly, without any calcium bound, there are more than 20,000 more calmodulin molecules in the $T$ than in the $R$ state. With one calcium ion bound, there are still $\approx 80 \times$ more calmodulin molecules in the $T$ than in the $R$ state. The equilibrium shifts toward the $R$ state when two or more calcium ions are bound: with three calcium ions bound, there is $\approx 800 \times$ as much calmodulin in the $R$ state as in the $T$ state, and when fully saturated, there are nearly 200,000 calmodulin molecules in the $R$ state for each calmodulin molecule in the $T$ state. The shift of equilibrium from $T$ to $R$ state with three or more calcium ions bound can also be seen from the free energy diagram (see Fig. 1).

**Altered Affinity for Calcium if Calmodulin Is Bound to a Target.** It has been reported (18–20) that the apparent affinity of calmodulin for calcium increases if calmodulin is bound to a target. To reproduce this effect, we simulated calcium binding to $2 \times 10^{-7}$ M calmodulin at varying calcium concentrations in the absence of target and in the presence of CaMKII. We also performed simulations on two reduced models, representing only the $R$ state or only the $T$ state of calmodulin, respectively. Results are shown in Fig. 4. The fractional occupancy curve illustrates how, with increasing calcium concentration, the calmodulin population shifts from mostly $T$ state to mostly $R$ state, with the corresponding affinities for calcium. The presence of target further

| Table 1. Apparent sequential association constants (molar) for our model, and comparison to the models of Crouch and Klee (16), Porumb (30), and to other experimental reports (18, 28–30) and data reviews (28) |
|---|---|---|---|---|
| Our model | Crouch and Klee (16) | Porumb (30) | Reported range |
| $K_1$ | $4.7 \times 10^5$ | $3 \times 10^5$ | $2.0 \times 10^5$ | $1.16 \times 10^5 \pm 1.7 \times 10^6$ |
| $K_2$ | $3.7 \times 10^5$ | $8.6 \times 10^5$ | $8.9 \times 10^5$ | $1.4 \times 10^5 \pm 8.9 \times 10^5$ |
| $K_3$ | $1.5 \times 10^5$ | $1.2 \times 10^5$ | $3.2 \times 10^4$ | $2.86 \times 10^4 \pm 2.9 \times 10^6$ |
| $K_4$ | $3.7 \times 10^4$ | $4.5 \times 10^4$ | $1 \times 10^5$ | $1.7 \times 10^{35} \pm 1.12 \times 10^5$ |

*Ref. 28.
†Ref. 30.
§Ref. 18.
¶Ref. 29.
Differential Activation of CaMKII and PP2B. An intriguing feature of calmodulin is its ability to differentially activate PP2B or CaMKII, depending on calcium input. It is known that PP2B has a much higher affinity for calmodulin (31) than CaMKII (32). However, although PP2B has been detected in PSD fractions (reviewed in ref. 33), CaMKII is far more abundant and, in fact, constitutes a major PSD protein (34, 35). Together with our allosteric model, this information can explain the differential activation of PP2B at low calcium concentrations and of CaMKII at high calcium concentrations.

As Fig. 5 shows, we can reproduce a differential activation of PP2B and CaMKII in our model. At subsaturating calcium concentrations, PP2B becomes preferentially activated due to its high affinity for the R state. PP2B binding stabilizes the R state and thus increases the apparent affinity of calmodulin for its targets. At higher calcium concentrations, more calmodulin in the R state is available and can bind to CaMKII. Because of the abundance of CaMKII, there is more active CaMKII than PP2B in absolute terms.

Discussion

Induced-Fit Models vs. Thermal Equilibrium Models. Both experimentalists (37) and theoreticians (38) have directly related calmodulin activity to calcium concentration, often by using a Hill function (39). This allows for some calmodulin activity even at low calcium concentrations. Furthermore, together with the facts that calmodulin affinity for PP2B (31) is higher than for CaMKII (32) and that CaMKII concentration in the PSD (40) is higher than PP2B concentration (41), it can explain why PP2B is preferentially activated at lower calcium concentrations. Models of this type do not allow for the possibility that the different binding sites might have different affinities for calcium. Recent experiments on recombinant calmodulin with site-directed mutations, however, suggest that the N-terminal binding sites have different affinities than the C-terminal binding sites (18). Finally, use of a Hill function to compute calmodulin activity as a function of calcium concentration is purely phenomenological and does not provide an explanation for how calcium binding relates to calmodulin activity.

Many investigators measuring calcium binding to calmodulin (16, 18, 28, 42) have used an Adair function (43), or a modified version thereof, to describe calmodulin binding to calcium. This approach allows for different microscopic affinities for the four binding events and thus for detailed models of calcium binding to calmodulin. These models do not, however, explain the transition of calmodulin between closed (“inactive”) and open (“active”) conformations.

Another way to model calmodulin action is by assuming a sequential mechanism, where calmodulin first binds four calcium ions and only then becomes activated (44, 45). These models are useful approximations of how calmodulin activates CaMKII, but they fail to account for important aspects of calmodulin function, all of which can be explained by our model. A sequential model fails to explain why calmodulin can activate PP2B or CaMKII when bound to less than four calcium ions (17, 18). Some models assume that calmodulin can open PP2B with two, three, or four calcium ions bound but requires four calcium ions bound to activate CaMKII (49). Although these models are decent phenomenological approximations for modeling the differential activation of CaMKII and PP2B, they do not provide a satisfactory mechanism by which calmodulin with three bound calcium ions activates PP2B, but not CaMKII. By postulating an equilibrium between inactive and active states that is shifted toward the active state by calcium, our model allows for calmodulin activity even when bound to less than four calcium ions. One can see the extent of this by looking at what fraction of subsaturated calmodulin becomes activated. As shown above, ≈75% of calmodulin molecules are active even when only two molecules of calcium are bound. This can explain experiments with mutant calmodulin showing that even when the two of the calcium-binding sites are ablated, calmodulin can still activate CaMKII to a certain extent (18). It is also consistent with experimental data showing that conformational transitions exist in apo calmodulin (46), and that both apo calmodulin and calmodulin bound to only one calcium ion exist in open and closed states (47). Finally, calmodulin open structures have been found where only one head was populated by calcium (48).

Because purely sequential models do not allow for subsaturated calmodulin binding to its target, they necessarily fail to account for the fact that the affinity of calmodulin for calcium increases once calmodulin is bound to a target (18–20). In our model, subsaturated calmodulin can bind to its targets. The apparent increase in calcium upon binding to targets arises from
the fact that targets act as allosteric activators, drawing the equilibrium toward the high-affinity \( R \) state.

Note that a simple model based on thermal equilibrium has been proposed before (50), although based on sequential bindings of calcium. Furthermore, the author did not try to estimate parameters using experimental information or to validate the model.

**Model Characteristics.** The association constants we determined from the simulation result by fitting it with an Adair-type equation (43) are the observable sequential association constants for the first, second, third, and fourth binding events. It is important to note, however, that our model does not assume a fixed order for calcium binding to the different binding sites. Rather, because the first calcium ion can bind to any one of four binding sites in either state, \( K_1 \) is a combination of the microscopic \( K_i^a \) and \( K_i^b \) values for each of the sites in each of the states. The apparent sequential dissociation constants used in our model follow an Adair-type (43) framework as used, for instance, by Crouch and Klee (16) and Porumb (30). They are, in principle, Adair constants, except for a slight difference in nomenclature: \( K_i \) according to Adair (43) corresponds to \( K_1 \times K_2 \) according to Crouch and Klee (16) and Porumb (30), and so on.

It is important to note that the conformation of calmodulin which we call the \( T \) state is not necessarily exactly identical to the reported apo structure (14) of calmodulin. Rather, the \( T \) state represents a collection of structures that may differ somewhat in the conformation of the calcium-binding sites, but whose overall structure resembles that of apo calmodulin. The existence of an ion-bound form that resembles the apo conformation has recently been established (51). Likewise, the \( R \) state is a collection of structures that resemble the reported open structure of active calmodulin (52, 53). Asymmetric forms of calmodulin with one lobe in an open state and one head in a closed state have been reported in the presence of some targets (54). However, the binding mechanism of these targets differs from that of CaMKII and PP2B, where both lobes bind to the target.

**Lisman Hypothesis.** The model proposed here explains how different amounts of calcium can trigger the activation of PP2B or CaMKII and thus provides support for the Lisman hypothesis (3) on a molecular level. The question of how different frequencies of calcium signals lead to differential activation of PP2B or CaMKII is not addressed in the model. It has been suggested, however, that, at least under some conditions, high frequencies of calcium input result in high local concentrations of calcium, whereas low calcium frequencies result in moderate local calcium concentrations in the spine (44, 55, 56). In addition, calcium frequency has a direct impact on CaMKII, because of the requirement for two adjacent subunits to be active for autophosphorylation at threonine residue 286, which confers sustained activity (57–59). Other factors the model does not account for include variations in the subcellular localization of PP2B (reviewed in ref. 60) and CaMKII (61) and the inhibitory effect of PP2B on CaMKII (reviewed in ref. 10). The latter effect, if included, would increase the window of calcium concentrations at which PP2B is preferably activated, enhancing the distinction between PP2B and CaMKII activation. We believe, however, that our model provides a valid and useful biophysical basis on which to develop further models of synaptic plasticity mechanisms.

**Note added in proof (64).** Experimental support for our equilibrium model of calmodulin function has been recently published by Gsponer et al. (64).

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